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Inviscid Wing-Tip Vortex Behavior Behind Wings in Close Formation Flight

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Nomenclature

b_H	=	nondimensionalized half-span of the hitchhiker aircraft by the half-span of the hitchhiker aircraft, Eq. (1)
b_M	=	half-span of the mothership divided by the half-span of the hitchhiker aircraft
dt	=	time step, s
dy	=	distance between the mothership and the hitchhiker divided by the half-span of the hitchhiker aircraft
dz	=	relative height between the mothership and the hitchhiker divided by the half-span of the hitchhiker aircraft
N	=	total number of elements representing each wing
$(u, v)_j^{HH}$	=	induced velocity at a wake vortex j on the hitchhiker aircraft by other vortices on the hitchhiker aircraft
$(u, v)_j^{HM}$	=	induced velocity at a wake vortex j on the hitchhiker aircraft by vortices from the mothership aircraft
$(u, v)_j^{MH}$	=	induced velocity at a wake vortex j on the mothership aircraft by vortices on the hitchhiker aircraft
$(u, v)_j^{MM}$	=	induced velocity at a wake vortex j on the mothership aircraft from other vortices on the mothership aircraft
x	=	downstreamwise distance from a wing's trailing edge
Γ_0^H	=	maximum circulation on the hitchhiker for an elliptic load distribution
Γ_0^M	=	maximum circulation on the mothership for an elliptic load distribution
δ	=	smoothing factor, 0.1

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Introduction

COMPOUND-AIRCRAFT-TRANSPORT (CAT) flight involves wing-tip-docked or close-formation flight between two or more aircraft to exploit mutual aerodynamic interaction benefits.[1] The larger aircraft is called the mothership, and the smaller aircraft is the hitchhiker. The upwash from the trailing vortex system of the mothership aircraft reduces the energy necessary to maintain flight by the hitchhiker. Magill et al.[1] performed a wind-tunnel test for three CAT flight configurations: wing-tip-docked, close formation, and towed formation. They found that the close-formation configuration had the largest lift-to-drag-ratio performance benefit for the hitchhiker aircraft.

Vortex lattice methods (VLM) are typically used for aerodynamic modeling of aircraft in formation flight. They can be used to determine the optimum downwash or spanload distributions[2,3] and the associated minimum drag, individually and for the entire system.[1,4–6] Modified horseshoe vortex models have been developed for use in adaptive or optimal control of aircraft in formation flight.[7,8] However, the predicted results using both the classical VLM methods and the modified horseshoe vortex models show a discrepancy between the measured data and the predicted results when the two wings are close to each other.[1,4,7] The discrepancy is attributed to the assumption of a flat wake[1] and the flow separation at the hitchhiker's wing tip induced by upwash from the mothership.[4]

Recently, Wang and Mook[9] showed that there is a possible wake-wake interaction by calculating the wake shapes behind wings in close formation flight using an unsteady vortex lattice method with a deforming wake. However, most results have been presented assuming the flat-wake model. The shape of the cross section of the three-dimensional vortex sheet behind the wing resembles an unsteady two-dimensional vortex sheet.[10] Discrete vortex methods have been applied to investigate the dynamics of the evolution of unsteady two-dimensional vortex sheets to understand the large-scale behavior of the roll up of aircraft trailing vortices (see Sarpkaya[11] for a comprehensive review).

The objective of the present work is to investigate the wing trailing vortex system for wings in close formation flight using a discrete vortex method, including wake roll up.[12] The wing bound vortex system is approximated by lifting lines with elliptic load distributions. The trailing wake sheets from the lifting lines are represented by point vortices that deform freely by the assumption of a force-free position during the simulation.

Discrete Vortex Method

The vortex-sheet approach used here is appropriate at high Reynolds numbers because the wake vortices are confined to a thin region and the viscous effects are negligible outside of this region. The span of the wing is discretized as N elements. The trailing vortex sheet is represented by rectilinear vortex filaments that are initially located at the $\frac{3}{4}$ point of the element.[12] In a cross plane normal to the vortex sheet, the positions of the vortex filaments are modeled as point vortices. The velocity components that are induced at the point (y_j, z_j) by a point vortex of strength Γ_i that is located at (y_i, z_i) are[12]

$$\boldsymbol{w}_j = \frac{\Gamma_i}{2\pi}\sum_{i=1}^{N}\left[\frac{(z_j - z_i)}{r_{ji}^2 + \delta^2}\boldsymbol{i} - \frac{(y_j - y_i)}{r_{ji}^2 + \delta^2}\boldsymbol{j}\right] \tag{1}$$

where δ is a smoothing factor.[13] Because the wake is force free, the evolution of each vortex is found by tracing the point vortices and moving the wake at the local flow velocity using an Euler convection scheme:

For a point vortex in the mothership:

$$(\Delta y, \Delta z)_j^M = \left[(u, v)_j^{MM} + (u, v)_j^{MH}\right] \mathrm{d}t \tag{2a}$$

For a point vortex in the hitchhiker:

$$(\Delta y, \Delta z)_j^H = \left[(u, v)_j^{HM} + (u, v)_j^{HH}\right] \mathrm{d}t \tag{2b}$$

Results

Figures 1 and 2 show the trailing vortex sheets when the mothership and hitchhiker aircrafts are flying close to each other. As shown in Fig. 1, the wing-tip vortices generated at the wing tips facing each other move up when the two wings move close to each other. The two outboard wing-tip vortices maintain their positions, essentially unaffected by the other aircraft. When the span of the mothership is twice as large as the hitchhiker (in Fig. 2), the wing-tip vortex of the mothership, with the larger circulation, pulls up the tip vortex of the hitchhiker.

Figure 3 shows the effect of relative height between the mothership and the hitchhiker. For a fixed value of $dy = 0.1$, the small relative height has the effect of moving up wake vortices behind both the mothership and the hitchhiker. However, the asymmetry in the wake vortices behind the hitchhiker is larger than that of the mothership for a small relative height. Though not presented here, the asymmetry in the wake vortices will result in a rolling moment for both vehicles.

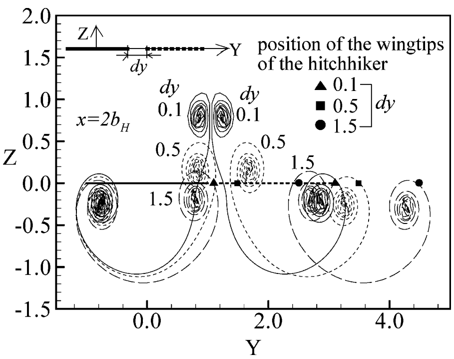

Fig. 1 Effect of gap on the positions of the wing-tip vortices behind elliptically loaded wings in close formation flight, when two wings have the same wing loadings ($\Gamma_0^M = \Gamma_0^H = 1.0$, $b_M = 1.0$, $dz = 0.0$, $dt = 0.01$, $N = 500$).

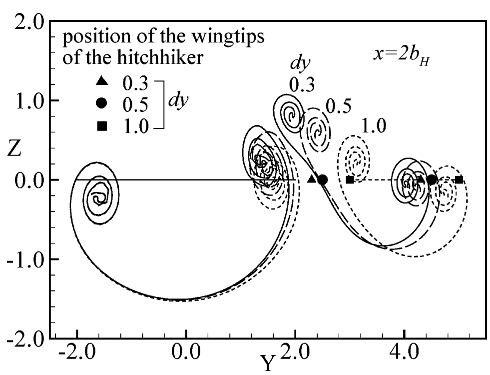

Fig. 2 Effect of gap on the positions of the wing-tip vortices behind elliptically loaded wings in close formation flight, when the mothership has larger wing loadings than the hitchhiker ($\Gamma_0^M = 2.0$, $\Gamma_0^H = 1.0$, $b_M = 2.0$, $dz = 0.0$, $dt = 0.01$, $N = 400$).

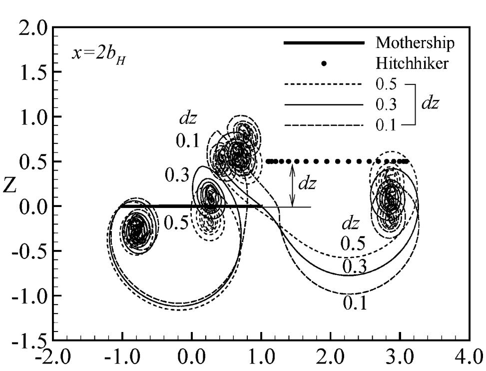

Fig. 3 Effect of the relative height between the aircraft ($\Gamma_0^M = \Gamma_0^H = 1.0$, $b_M = 1.0$, $dy = 0.1$, $dt = 0.01$, and $N = 1000$).

Conclusions

This study demonstrates the inviscid near-wake behavior behind aircraft flying in close formation. The prediction of the wake behavior shows that the distance between the wings has the effect of moving the wake vortices up behind wing tips facing each other. The relative heights between aircraft cause a large asymmetric evolution of wake vortices, which will result in a change of aerodynamic characteristics. Although the present results are obtained based on potential flow theory, it can be concluded that the prediction of the wake roll-up behavior behind aircraft in formation flight might be of considerable importance in the prediction of the aerodynamic performance.

Acknowledgment

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References

[1] Magill, S. A., Schetz, J. A., and Mason, W. H., "Compound Aircraft Transport: A Comparison of Wingtop-Docked and Close-Formation Flight," AIAA Paper 2003-607, Jan. 2003.

[2] Iglesias, S., and Mason, W. H., "Optimum Spanloads in Formation Flight," AIAA Paper 2002-0258, Jan. 2002.

[3] Frazier, J. W., and Gopalarathnam, A., "Optimum Downwash Behind Wings in Formation Flight," *Journal of Aircraft*, Vol. 40, No. 4, 2003, pp. 799–803.

[4] Blake, W. B., and Gingras, D. R., "Comparison of Predicted and Measured Formation Flight Interference Effects," AIAA Paper 2001-4136, Aug. 2001.

[5] Blake, W. B., and Multhopp, D., "Design, Performance and Modeling Considerations for Close Formation Flight," AIAA Paper 98-4343, Aug. 1998.

[6] Wagner, C. G., Jacques, L. D., Blake, W. B., and Pachter, M., "An Analytical Study of Drag Reduction in Tight Formation Flight," AIAA Paper 2001-4075, Aug. 2001.

[7] Venkataramanan, S., Dogan, A., and Blake, W. B., "Vortex Effect Modelling in Aircraft Formation Flight," AIAA Paper 2003-5385, Aug. 2003.

[8] Binetti, P., Ariyur, K. B., Krstic, M., and Bernelli, F., "Formation Flight Optimization Using Extremum Seeking Feedback," *Journal of Guidance, Control, and Dynamics*, Vol. 26, No. 1, 2003, pp. 132–142.

[9] Wang, Z., and Mook, D., "Numerical Aerodynamic Analysis of Formation Flight," AIAA Paper 2003-610, Jan. 2003.

[10] Pullin, D. I., "The Large-scale Structure of Unsteady Self-similar Rolled-up Vortex Sheets," *Journal of Fluid Mechanics*, Vol. 88, Pt. 3, 1978, pp. 401–430.

[11] Sarpkaya, T., "Computational Methods with Vortices-the 1988 Freeman Scholar Lecture," *Journal of Fluids Engineering*, Vol. 111, No. 1, 1989, pp. 5–52.

[12] Han, C., and Cho, J., "Unsteady Trailing Vortex Evolution Behind a Wing in Ground Effect," *Journal of Aircraft*, Vol. 42, No. 2, 2005, pp. 429–434.

[13] Krasny, R., "Computation of Vortex Sheet Roll-up in the Trefftz Plane," *Journal of Fluid Mechanics*, Vol. 184, 1987, pp. 123–155.